

IARS 2009 SUMMER SCHOOL IN MATHEMATICAL PHYSICS

Lecture 1 (2 × 1 hour): Sets, equivalence relations, functions, image and inverse image of a set under a function, one to one and onto functions, composition of functions, inverse function, equivalent sets; binary operations and groups, group homomorphism and isomorphisms, group structure; vector spaces, span and linear independence, basis and dimension, linear operators, vector space isomorphisms, basis transformations in a finite dimensional vector space

Lecture 2 (2 × 1 hour): Dual space and dual basis, tensors on a finite dimensional vector spaces, symmetric and antisymmetric tensors, tensor and wedge product, Grassmann algebra of a vector space; inner product spaces, norm, metric operator and its role in associating a unique covector to each vector

Lecture 3 (2 × 1 hour): Metric spaces, open, closed, and bounded subsets, continuous functions between metric spaces and sequence convergence, isometries and metric space isomorphism; topological spaces, metric, discrete, and trivial topology, subspace of a topological space, product topology, continuous functions between topological spaces, homeomorphisms, topological equivalence and invariants, connectedness, compactness.

Lecture 4 (2 × 1 hour): Bases for a topology, first and second countable topological spaces, Hausdorff spaces; Euclidian spaces, derivative of a function mapping between Euclidean spaces, inverse and implicit function theorems; locally Euclidean topological spaces, topological manifolds, coordinate charts and atlases

Lecture 5 (2 × 1 hour): Product, C^n , and smooth manifolds, C^n functions between manifolds, C^n diffeomorphisms, Diffeomorphism equivalent C^n manifolds, compatible C^n atlases and C^n structures on a topological manifold, diffeomorphism equivalent C^n manifolds with different C^n structures; local coordinate systems and transformations, smooth curves, tangent vectors, tangent spaces of a smooth manifold.

Lecture 6 (2 × 1 hour): Tangent vectors as derivations, cotangent space, tensor fields, differential forms and the exterior algebra on a smooth manifold, local coordinate representation and transformations of vector fields, differential forms and tensor fields on a manifold.

Lecture 7 (2 × 1 hour): Immersions, embeddings, submanifolds, Frobenius theorem.

Lecture 8 (2 × 1 hour): Exterior derivative and its algebraic and analytic formulations. Contraction operation.

Lecture 9 (2 × 1 hour): Diffeomorphisms, 1 parameter subgroups, flow equations, definition of the Lie derivative. Algebraic and analytic aspects of Lie derivative.

Lecture 10 (2 × 1 hour): Definition of the metric, geodesic distance, Hopf Rinow theorem. Hyperbolic plane as an example. Isometries.

Lecture 11 (2 × 1 hour): Covariant derivative, parallel transport, Cartan's moving frames.

Lecture 12 (2 × 1 hour): Curvature, torsion, Cartan's structure equations, symmetries of the curvature tensor, Bianchi identity.

Lecture 13 (2 × 1 hour): Frobenius theorem in terms of differential forms. Sectional curvature. Jacobi equation.

Lecture 14 (2 × 1 hour): Riemann normal coordinates, some theorems of Riemannian geometry.

Lecture 15 (2 × 1 hour): Extrinsic and intrinsic curvature of submanifolds.

Lecture 16 (2 × 1 hour): Integration on manifolds. de Rham cohomology.

**Applications of differential geometry in general relativity
(approximately 10 lectures, 2 hours each)**

1. Special Relativity and Flat Spacetime

Lorentz Transformations

Vectors and Dual Vectors (One-Forms)

Tensors and Their Manipulations

Energy-Momentum Tensor

2. Manifolds

Gravity as Geometry

Manifolds

Vectors and Tensors Again

The Metric

3. Curvature

Covariant Derivatives

Parallel Transport and Geodesics

The Riemann Tensor and Its Properties

Geodesic Deviation

4. Physics in Curved Spacetime

Einstein Equation

The Schwarzschild Solution